TIME-DOMAIN METHOD OF LINES APPLIED TO A PARTIALLY FILLED WAVEGUIDE*

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ABSTRACT

A new time-domain method for the analysis of guided wave propagation and scattering is developed in which an analytical process is incorporated along one of the three dimensions in space, so that the problem is effectively reduced to a two-dimensional one. A simple numerical example is presented as a demonstration of the new method.

INTRODUCTION

The time-domain analysis of microwave planar transmission structures provides an alternative to the frequency domain approach, and is also useful for studying the behavior of pulsed signals in structures such as high speed digital circuits. A typical time-domain analysis requires discretization of a three-dimensional space into a three-dimensional mesh. Usually, a large computer storage and a long computation time are required. An additional problem of these methods is the difficulty in handling open boundaries.

OUTLINE OF THE PROPOSED METHOD

The proposed method originates from the fact that most of the discontinuities appearing in the planar transmission structures are located on the substrate surface and the space below and above this surface is uniform and homogeneous. We wish to solve the problem by discretizing only in a two dimensional surface on the substrate where the discontinuity is located. This is possible if the wave scattering information in the direction perpendicular to the substrate surface is available analytically. The proposed method actually incoporate this process. The method is somewhat similar to the frequency domain analysis called the method of lines [1].

The method entails discretization of the structure by a number of lines perpendicular to the substrate surface as shown in Fig.1. At the specified time, the field distribution at each intersection of these lines with the substrate surface is calculated by Maxwell's equations discretized only in the x and z directions that are parallel to the substrate surface. The field information in the y direction is obtained analytically at each point and time. This information can be found from the inverse Fourier transform of the solution of the frequency domain Helmholtz equation in the y direction.
One may wonder as to what is happening to the wave scattering phenomena that is taking place everywhere in the waveguide, not only on the substrate surface. This question is natural, because in other time domain methods the electromagnetic fields at one mesh point interact with those at all six neighboring mesh points in x, y and z directions. In the proposed method, the fields at any point on one discretization line do not appear to interact with those on a similar point on another line. It should be emphasized that this is not the case. As we will see shortly, a spatial transformation is introduced by which the field as a function of (discretized) x and z is transformed to another discretized quantity which contains the field quantities at all x and z values. The analytical information in the y direction is then applied to this transformed quantity. Since analytical expressions are used for the field variation in the y direction, this method can easily handle the case where the top wall is removed, whereby the structure is open in the y direction.

A SIMPLE EXAMPLE FOR A TWO-DIMENSIONAL PROBLEM

Let us consider a simple two-dimensional structure as a test case. The formulation for such a structure is simplified, yet it contains all essential features of the proposed method. As shown in Fig.2, the problem is a partially filled rectangular waveguide excited by an electric field, E_z, infinite in length and uniform in the z (axial) direction. The problem is now a two-dimensional one. This problem corresponds to finding the cutoff frequencies of various TM modes in the frequency domain [2]. Such information can be extracted from the time domain data.

Because of the excitation, only E_z, H_x and H_y exist and $\partial/\partial z = 0$. The time-domain equations are discretized in the x-direction only.

$$-\mu\, \partial[H_x] / \partial t = \partial[E_z] / \partial y \tag{1a}$$

$$-\mu\, \partial[H_y] / \partial t = [D_x{}^e]\, [E_z] / \Delta x \tag{1b}$$

$$\varepsilon(y)\, \partial[E_z] / \partial t = [D_x{}^h]\, [H_y] / \Delta x - \partial[H_x] / \partial y \tag{1c}$$

$$[D_{xx}{}^e][E_z] / (\Delta x)^2 + \partial^2[E_z] / \partial^2 y - \mu\varepsilon(y)\, \partial^2[E_z] / \partial^2 t = 0 \tag{1d}$$

where $[D_x{}^e]$, $[D_x{}^h]$, $[D_{xx}{}^e]$ are difference operators in which the side wall boundary condition is incorporated [1]. The variables $[E_z]$, $[H_x]$ and $[H_y]$ are the column vectors representing the fields along each line and are functions of y and t. Since $[D_{xx}{}^e]$ is a real symmetric matrix, there exists a real orthogonal matrix $[T_x^e]$ that transforms $[D_{xx}^e]$ into a diagonal matrix $[d_{xx}^e]$. We can now transform $[E_z]$, etc. into a transform $[E_z] = [T_x^e]^t[E_z]$ etc.where the superscript t stands for transpose. The transform of the equation (1-d) is

$$(1/\Delta x)^2[d_{xx}^e][E_z] + \partial^2[E_z] / \partial^2 y - \mu\varepsilon(y)\, \partial^2[E_z] / \partial^2 t = 0 \quad (2)$$

Notice that (2) is a set of uncoupled partial differential equations, that is, it can be solved independently along the i-th line. Using the separation of variable technique, one can obtain a typical Sturm-Liouville differential equation for the y dependant solution. The solution for the i-th line is

$$E_{zi}(y, t) = \Sigma_n (A_{ni} \cos \omega_{ni}t + B_{ni} \sin \omega_{ni}t) \sin K_{1ni}(b-y) \quad \underline{\text{for region I}}$$

$$\Sigma_n (A_{ni} \cos \omega_{ni}t + B_{ni} \sin \omega_{ni}t) (\sin K_{1ni}d / \sin K_{2ni}h) \sin K_{2ni}y \quad \underline{\text{for region II}} \quad (3)$$

where K_{1ni}, $K_{2ni,}$ and ω_{ni} are determined by the characteristic transcendental equation

$$K_{1ni} \cos K_{1ni}d \sin K_{2ni}h + K_{2ni} \sin K_{1ni}d \cos K_{2ni}h = 0 \quad (4)$$

$$[(K_{1ni})^2 - d_{xxi}^e / (\Delta x)^2] / \mu\varepsilon_1 = [(K_{2ni})^2 - d_{xxi}^e / (\Delta x)^2] / \mu\varepsilon_2 \quad (5)$$

$$\omega_{ni}^2 = [(K_{1ni})^2 - d_{xxi}^e / (\Delta x)^2] / \mu\varepsilon_1 \quad (6)$$

From this point on, there are basically two approaches. By knowing the initial conditions for E_z and its time derivative, one can find A_{ni} and B_{ni}. Then, the solution at any point at any time can be extracted from the inverse transform via $[E_z(y, t)] = [T_x^e][E_z(y, t)]$.

An alternative method is an application of the time stepping procedure. From the initial condition for E_z, one can find A_{ni} at time t = 0 in the equation (3) which will be called A_{ni}^N with N = 0. With the causality condition, the transforms of (1a) ~ (1c) can be discretized in time as a time stepping iteration. Expressing the solution (3) in the form

$$E_{zi}^N(y) = \Sigma_n A_{ni}^N \sin K_{1ni}(b-y) \quad \underline{\text{in region I}}$$

$$\Sigma_n A_{ni}^N (\sin K_{1ni}d / \sin K_{2ni}h) \sin K_{2ni}y \quad \underline{\text{in region II}} \quad (7)$$

and a similar one for H_{xi} and H_{yi} at the time (N + 1/2), one can implement a leap-flog type iteration scheme to calculate these coefficients. The real field at $y = y_o$ at the N-th time step can be obtained by invoking the inverse transformation as described above to $[E_z]^N$.

RESULTS AND DISCUSSION

The accompanying figures are results of sample calculations. Figure 3 depicts the E_z field distributions in the waveguide cross section (x-y plane) at each time step after a pulsed E_z excitation is imposed at t = 0 at the center of the cross section. Figure 4 shows the spectrum of the time signal for E_z where the waveguide cutoff frequencies are represented by the peaks. The results differ by less than one percent from the analytical values.

It should be noted that the two methods described above should be equivalent in principle. However, each has advantages and disadvantages. For instance, if one deals with a time-spread excitation, the time stepping method would be simpler to implement. Otherwise, the first method is more efficient as long as only the results at a particular time are of interest. However, if any frequency domain information is needed, the time history needs to be found. The first method needs to be used at many time instances. The time stepping method automatically generates the time history. Hence, the amount of computation would be about the same. In many cases, there are ambiguities in finding the time derivative of the initial condition. In such instances, it may be simpler to use time stepping to generate the time history required. It should be noted that switching from one method to the other is quite possible.

CONCLUSIONS

A new time domain technique is presented in which an analytical process is incoporated along one of the spatial dimensions so that the dimensions of the problem are effectively reduced by one. The present approach can be used to calculate the cutoff frequency of the other planar transmission structures and can be extended to the propagation problems. It has a number of potential advantages over many other time domain methods. First, the method is believed to be efficient since much analytical processing is used. Second, the problem can handle open boundaries in the vertical (y) direction because of the analytical solutions used in y.

REFERENCES

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2. Hoefer,W.,"TheTransmission-Line Matrix Method - Theory and Applications," IEEE Trans., 1985, MTT-33, pp. 882-893.

* This work was sponsored by the Office of Naval Research under Contract N00014-79-C-0553.

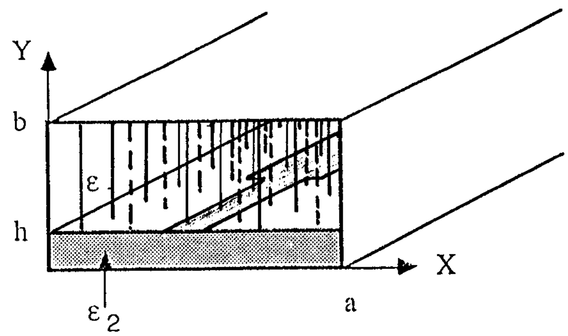

Fig. 1 Typical planar transmission line structure with a discontinuity and its discretization example for the analysis.

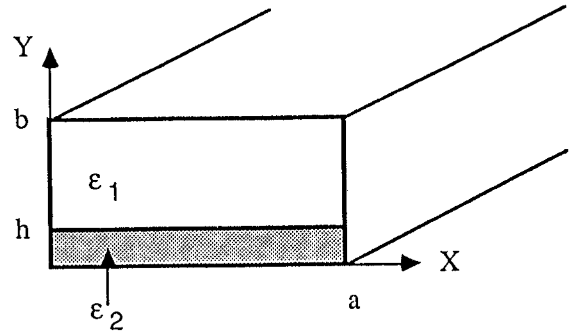

Fig. 2 Partially filled rectangular waveguide structure.

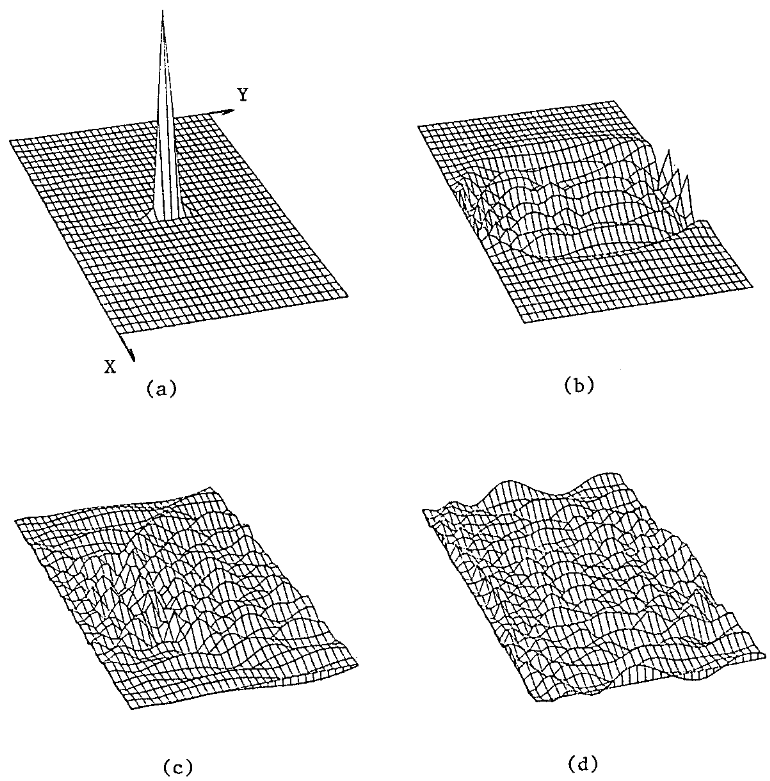

Fig. 3 Pictures of the distribution of E_z field at various times. (a = 2, b = 1, h = 0.2 [cm], $\varepsilon_1 = 1$, and $\varepsilon_2 = 3$)
(a) t = 0, (b) t = 20, (c) t = 40, and (d) t = 60 [pico-sec]

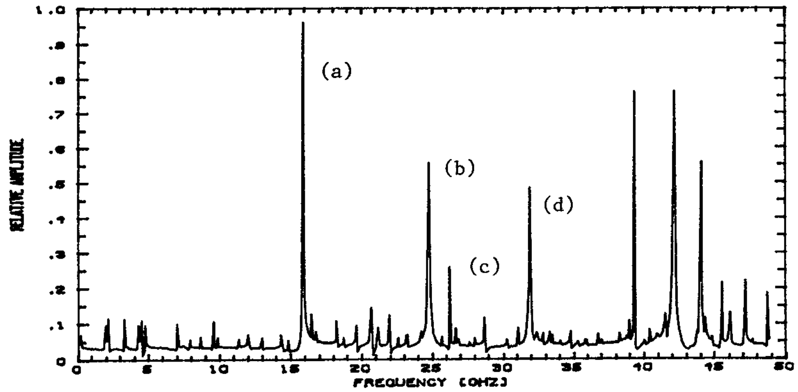

Fig. 4 Cutoff frequency spectrum for the structure shown in fig.2 (a = 2, b = 1, h = 0.2 [cm], $\varepsilon_1 = 1$, and $\varepsilon_2 = 3$)
(a) TM_{11}, (b) TM_{31}, (c) TM_{12}, and (d) TM_{32} etc.